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ON TERAELECTRONVOLT MAJORANA NEUTRINOS

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The issue of existence of Majorana neutrinos with masses of the order of TeV and substantial couplings is addressed. A general neutrino mass matrix M_ν with both features is constructed, however, the form of M_ν is constrained very much by severe relations among the elements of $m_{\rm D}$ and $M_{\rm R}$ sub-matrices of M_ν. These general relations follow from the perturbative construction of the light neutrino mass spectrum. To avoid such large correlations between low mass parameters in $m_{\rm D}$ and large mass parameters in $M_{\rm R}$, the Jeżabek–Sumino see-saw model of bi-maximal neutrino mixing adopted to the TeV scale and the issue of possible symmetries of the matrix M_ν are discussed. Results are supported by a few numerical examples which show directly the complexity of the problem.

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1. Introduction

In the see-saw scenario [1] light neutrinos with masses of the eV scale demand heavy neutrino masses to be of the order of 10^9 GeV at least. The diagonalization of the neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & m_{\rm D} \\ m_{\rm D}^{\rm T} & M_{\rm R} \end{pmatrix}, \tag{1}$$

in the case $m_{\rm D} \ll M_{\rm R}$ (we assume, without lose of generality, that both matrices are of dimension 3), gives three light m_ν and three heavy m_N masses of neutrinos. To a good approximation their scale is given by [2]

$$m_\nu \simeq -m_{\rm D}^{\rm T} M_{\rm R}^{-1} m_{\rm D}, \tag{2}$$

$$m_N \simeq M_{\rm R}\,. \tag{3}$$

We can see that to get light neutrino masses of the order of electronvolts one needs elements of the matrix $M_{\rm R}$ larger than 10^9 GeV (elements of the $m_{\rm D}$ matrix are typically taken to be of the order of 1 GeV, scale of masses of charged leptons). Smaller masses of light neutrinos demand even larger than $M_{\rm R}$.

It is clear that heavy neutrino states exhibit huge masses. Moreover, their couplings to the ordinary matter are negligible, namely, if $m_{\rm D} \ll M_{\rm R}$, the Light-Heavy (LH) neutrino mixing matrix $U^{\rm LH}$, which is a part of the full unitary matrix U diagonalizing M_ν $\left(U^{\rm T} M_\nu U = m_{\rm diag} \equiv {\rm diag}\left[m_i, M_i\right]\right)$ defined as

$$U = \begin{pmatrix} (U^{\rm Ll})^* & (U^{\rm LH})^* \\ U^{\rm Rl} & U^{\rm Rh} \end{pmatrix} \tag{4}$$

exhibits very small elements

$$U^{\rm LH} \sim m_{\rm D} M_{\rm R}^{-1} \ll 1. \tag{5}$$

In Eq. (4) the $U^{\rm Ll}$ sub-matrix is responsible for the neutrino mixing in the light sector, while the sub-matrix $U^{\rm Rh}$ describes neutrino mixings in the heavy neutrino sector. More details can be found for instance in [3,4]. If we look now into the form of the SM purely left-handed charged current written out in the mass eigenstates basis [4] (ν_i and N_i corresponds to light (heavy) neutrino mass states m_i (M_i), respectively)

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} \left[\sum_{i=1}^{3} \overline{\nu}_i \left(U^{\rm Ll}\right)_{il} \gamma^\mu P_{\rm L} l W_\mu^+ + \sum_{i=1}^{3} \overline{N}_i \left(U^{\rm LH}\right)_{il} \gamma^\mu P_{\rm L} l W_\mu^+ \right] + {\rm h.c.}\,, \tag{6}$$

it is obvious that effects from the heavy neutrino sector on processes with charged currents are completely unimportant (the same is true for the neutral current interactions [4]). This is a typical situation when the see-saw mechanism is explored. However, from experimental data we only know, that neutral leptons with masses below around $\mathcal{O}(10^2)$ GeV and with the typical weak neutrino coupling strength g are excluded [5]. There is no direct information on heavier neutral particles. From global fits to the data some bounds on the mixings of heavy neutrinos have been obtained [6]

$$\sum_N \left| (U^{\rm LH})_{Ne} \right|^2 \;\leq\; 0.0054, \tag{7}$$

$$\sum_N \left| (U^{\rm LH})_{N\mu(\tau)} \right|^2 \;\leq\; 0.0028\,(0.016). \tag{8}$$

These numbers are not negligible and effects of heavy neutrinos physics with the above mixings could be detected in future lepton (*e.g.* e^+e^- [7], e^-e^- [8–10]) or hadron [11] colliders. They can also influence processes generated by higher order corrections [14]. Finally, they may modify neutrino oscillation phenomena [15].

However, a natural question arises: is there any natural mechanism of heavy neutrinos creation with reasonably large mixings, or more precisely, what is the form of M_ν which would give such neutrino properties? Obviously, heavy neutrinos with TeV masses may lead to large $U^{\rm LH}$ elements, but too large masses of light neutrinos m_i would simultaneously arise. Usually symmetry arguments are invoked to show that it is possible to build up an appropriate form of M_ν [16]. There are also other scenarios which implement TeV neutrinos. They go in different ways, *e.g.*: charged Higgs bosons [17], bulk neutrinos or scalars [18], higher dimensional operators [19], naturally suppressed Dirac masses (through the presence of an extra scalar doublet) [20]. Whether any of these scenarios can be really assumed to be "natural" (and, maybe, used by nature) is an open question.

Here we would like to present and discuss the issue paying special attention to numerical results and their consequences. In the next section three typical examples of possible M_ν are given. In the first case masses of light neutrinos are too large, in the second M_ν is constructed to give exactly three massless neutrinos and finally appropriate masses of light neutrinos are obtained. At the same time masses of heavy neutrinos in the range ($100\,{\rm GeV} \leq M_i \leq 1\,{\rm TeV}$) and large $U^{\rm LH}$ elements which fulfill basic bounds Eqs. (7), (8) are obtained. Two Lepton Flavor Violating (LFV) processes, namely neutrino-less double beta and $\mu \to e\gamma$ decays are considered and the issue of large neutrino mixings in the light sector is discussed. Section 3 includes a discussion of a model with independent $m_{\rm D}$ and $M_{\rm R}$ matrices and the issue of possible symmetries of the matrix M_ν which could lead to the desired features of heavy neutrinos.

In the numerics we have to deal with huge differences of scales ($M_i/m_i \geq 10^{11}$) and the precision of calculations must be under control. To tackle this problem we used MATHEMATICA [21]. Simple cross checks of calculations are: unitarity of the U matrix Eq. (4) and recovering of M_ν to the same order of precision by the reverse relation $\left[U m_{\rm diag} U^{\rm T} = M_\nu\right]$. All results are obtained for the case of the 6×6 matrix M_ν.

2. TeV neutrino mass models with correlations among $m_{\rm D}$ and $M_{\rm R}$ matrix elements

Example I: Too large masses of light neutrinos

Let us start from the most general case where we only assume the naturalness of the $m_{\rm D}$ and $M_{\rm R}$ scales, without deeper insight into the relation among their elements. Let us take then the elements of the M_ν mass matrix Eq. (1) in the following form

$$m_{\rm D} = \begin{pmatrix} 0.8 & 1 & 0.9 \\ 1.5 & 0.5 & 0.1 \\ 0.7 & 1.2 & 2 \end{pmatrix} [\mathrm{GeV}], \tag{9}$$

$$M_{\rm R} = \mathrm{diag}\,(100, 150, 200)\ [\mathrm{GeV}]. \tag{10}$$

Without loss of generality, $M_{\rm R}$ has been taken in a diagonal form [22] with its elements close to the present experimental limit [5]. The result is

$$m_{\rm diag} \simeq \mathrm{diag}\,(3.5 \times 10^{-4}, 0.013, 0.062, 100, 150, 200)\,[\mathrm{GeV}]\,, \tag{11}$$

$$U^{\rm Ll} \simeq i \begin{pmatrix} 0.854 & -0.022 & 0.519 \\ -0.28 & 0.822 & 0.496 \\ -0.438 & -0.57 & 0.695 \end{pmatrix},$$

$$U^{\rm LH} \simeq \begin{pmatrix} 0.008 & -0.007 & -0.005 \\ 0.015 & 0.003 & -0.001 \\ 0.007 & 0.007 & -0.01 \end{pmatrix}. \tag{12}$$

In the above we restrict ourselves to the only interesting cases of light-light ($U^{\rm Ll}$) and light-heavy ($U^{\rm LH}$) neutrino mixing sectors. We can see that large LH neutrino mixings can be obtained but the masses of light neutrinos are too large. It is clear now, after neutrino oscillation data analysis that the mass of the heaviest of light neutrinos must be in the range [23]

$$0.04\,\mathrm{eV} \le m_3 \le 2.7\,\mathrm{eV}\,. \tag{13}$$

Let us note that both mixing angles and the mass spectrum still satisfy the see-saw relations Eq. (2) and (5).

Similarly it can be checked that the LH mixings could be larger with larger $m_{\rm D}$ elements. However, larger $m_{\rm D}$ would increase masses of light neutrinos (in agreement with Eq. (2)). Taking smaller $m_{\rm D}$ would on the other hand lead to appropriate m_i, but this time LH mixings would be completely out of interest. So, we need $m_{\rm D}$ elements to be at $\mathcal{O}(1)$ GeV level.

We can still try to change the ranks of $m_{\rm D}$ and $M_{\rm R}$ matrices. Then we can expect that additional light neutrinos appear. First, let us put the first entry in Eq. (10) as zero ($100 \to 0$) without other changes in the m_ν matrix elements. The result is

$$m_{\rm diag} \simeq {\rm diag}\,(3.8 \times 10^{-4}, 0.0228, 1.83, 1.85, 150, 200)\,[{\rm GeV}]\,. \qquad (14)$$

Second, let us also take the second entry in Eq. (10) as zero ($150 \to 0$). Then the mass spectrum is

$$m_{\rm diag} \simeq {\rm diag}\,(0.001, 0.783, 0.792, 2.33, 2.34, 200)\,[{\rm GeV}]\,. \qquad (15)$$

We can see that in the first case we get neutrinos with two masses of the order $m_{\rm D}/M_{\rm R}$, two of the order $m_{\rm D}$ and two of the order $M_{\rm R}$. In the second case these are: one mass of the order $m_{\rm D}/M_{\rm R}$, four masses of the order $m_{\rm D}$ and one of the order $M_{\rm R}$. This result fits to the discussion of the dependence between the rank of the $M_{\rm R}$ matrix and the scale of the obtained neutrino masses as given in [12].

We can also decrease the rank of the matrix $m_{\rm D}$. For that we take as an example

$$m_{\rm D} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.8 & 0.9 & 1 \end{pmatrix} [{\rm GeV}], \qquad (16)$$

$$M_{\rm R} = {\rm diag}\,(0, 150, 200)\ [{\rm GeV}]. \qquad (17)$$

The $m_{\rm D}$ matrix is of rank 1, the $M_{\rm R}$ matrix has the rank 2. The result is

$$m_{\rm diag} \simeq {\rm diag}\,(0, 0, 0.793, 0.807, 150, 200)\ [{\rm GeV}], \qquad (18)$$

$$U^{\rm Ll} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.704 \end{pmatrix}, \qquad U^{\rm LH} \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.71 & -0.009 & 0.005 \end{pmatrix}. \qquad (19)$$

Two massless neutrinos are obtained. Though the rank of the matrix $m_{\rm D}$ is 1, we get two neutrinos with masses of the order of $m_{\rm D}$ elements. We cannot further decrease the rank of the $m_{\rm D}$ matrix as heavy neutrinos would not mix with light states at all. Taking only one nonzero entry in the $M_{\rm R}$ matrix would not change the situation. The neutrino mass spectrum includes in this case three massless neutrinos, two massive neutrinos of the order of $m_{\rm D}$ and one heavy neutrino.

In this way we have shown that changing ranks of $m_{\rm D}$ and $M_{\rm R}$ matrices is not sufficient to get appropriate spectrum of neutrino masses (for further discussion of the meaning of $m_{\rm D}$ and $M_{\rm R}$ matrices of different ranks in general see-saw models see *e.g.* [13]).

We can see that there is no way around and we have to look for some relations among $m_{\rm D}$ and $M_{\rm R}$ elements of the M_ν matrix which would give appropriate masses of light neutrinos and TeV neutrinos with substantial mixings.

Example II: Three massless neutrinos

Let us take [11]

$$m_{\rm D} = m_{\rm D}^{(0)} + m_{\rm D}^{(1)}, \tag{20}$$

and assume that $m_{\rm D}^{(0)} \gg m_{\rm D}^{(1)}$.

Using Eq. (2) we get

$$\begin{aligned} m_\nu \;=\; & -m_{\rm D}^{(0)}\frac{1}{M_{\rm R}}m_{\rm D}^{(0){\rm T}} - \left(m_{\rm D}^{(1)}\frac{1}{M_{\rm R}}m_{\rm D}^{(0){\rm T}} + m_{\rm D}^{(0)}\frac{1}{M_{\rm R}}m_{\rm D}^{(1){\rm T}}\right) \\ & - m_{\rm D}^{(1)}\frac{1}{M_{\rm R}}m_{\rm D}^{(1){\rm T}}\,. \end{aligned} \tag{21}$$

The first term is the largest. It will give the largest contribution to $m_{\rm light}$. Let us demand that it is zero, *i.e.* $m_{\rm D}^{(0)}\,(1/M_{\rm R})m_{\rm D}^{(0){\rm T}} = 0$ and parameterize $m_{\rm D}^{(0)}$ in the most general way (elements of Eq. (22) can be complex)

$$m_{\rm D}^{(0)} = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} \equiv \begin{pmatrix} \alpha_i \\ \beta_i \\ \gamma_i \end{pmatrix}, \quad i = 1,\, 2,\, 3. \tag{22}$$

Then the following relation can be obtained ($M_{\rm R}$ has diagonal elements M_1, M_2, M_3)

$$\sum_i \begin{pmatrix} \frac{\alpha_i^2}{M_1} & \frac{\alpha_i\beta_i}{M_2} & \frac{\alpha_i\beta_i}{M_3} \\ \frac{\alpha_i\beta_i}{M_1} & \frac{\beta_i^2}{M_2} & \frac{\beta_i\gamma_i}{M_3} \\ \frac{\alpha_i\gamma_i}{M_1} & \frac{\beta_i\gamma_i}{M_2} & \frac{\gamma_i^2}{M_3} \end{pmatrix} = 0\,. \tag{23}$$

If $m_{\rm D}^{(1)} = 0$, it is a set of equations for relations among $m_{\rm D}^{(0)}$ and $M_{\rm R}$ elements which assure that three massless neutrinos are constructed. To show a numerical example, we will leave for a moment the most general case Eq. (22) and use $m_{\rm D}^{(0)}$ with the following texture

$$m_{\rm D}^{(0)} = \begin{pmatrix} \alpha_i \\ a\,\alpha_i \\ b\,\alpha_i \end{pmatrix}. \tag{24}$$

Then, instead of Eq. (23) we get only one condition

$$\frac{\alpha_1^2}{M_1} + \frac{\alpha_2^2}{M_2} + \frac{\alpha_3^2}{M_3} = 0\,. \tag{25}$$

Now we will use also the second important relation among heavy neutrino mass matrix elements which comes from the neutrino-less double beta experiments [24]

$$\left|\sum_i (U^{\rm LH})^2_{ie} \frac{1}{M_i}\right| = \omega^2\,, \tag{26}$$

where $\omega^2 < (2\text{–}2.8) \times 10^{-5}$ TeV^{-1}. There is no consensus concerning estimation of the ω parameter, nevertheless it appear that this relation is so severe that the possibility of heavy neutrinos detection in future colliders is drastically reduced [10] (see, however, [9, 25]). It can be checked that conclusions of the present paper do not change when $\omega = 0$ is assumed. Then relations (10), (25), (26) with $\alpha_1 = 3$ GeV allow to set α_2 and α_3. $U^{\rm LH}$ is taken in the form of Eq. (5), $a = 1$, $b = 0$. Then the matrix in Eq. (1) is fixed and, after its diagonalization, the set of physical neutrino parameters is obtained

$$m_{\rm diag} \simeq {\rm diag}\,(0, 0, 0, 100, 151, 201)\,[{\rm GeV}] \tag{27}$$

$$U^{\rm Ll} \simeq \begin{pmatrix} 0.998 & 0 & 0 \\ -0.004 & 0.998 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U^{\rm LH} \simeq \begin{pmatrix} 0.03 & -0.048\,i & 0.036 \\ 0.03 & -0.048\,i & 0.036 \\ 0 & 0 & 0 \end{pmatrix}. \tag{28}$$

The LH neutrino mixing is large (Eq. (28)) and fulfill Eqs. (7), (8). Three massless neutrinos are there. The spectrum of heavy states is as expected. In the SM massless neutrinos give diagonal $U^{\rm Ll}$. Here some small non-diagonal entries reflect the existence of heavy neutrino states. Crucial is Eq. (25). If we disturb it slightly then some of light neutrino states exhibit unacceptable values, *e.g.* if $\alpha_1 \to \alpha_1 + 10^{-6}$ GeV then (with the other parameters chosen just as before) we get

$$m_{\rm diag} = {\rm diag}\,(0, 0, 2 \times 10^{-7}, 100, 151, 201)\,[{\rm GeV}]\,. \tag{29}$$

Example III: Realistic light neutrino masses

To make our construction realistic two final issues must be addressed. The first is the exact spectrum of light neutrino masses and the second is their mixing pattern.

As for the light neutrino parameters let us try to recover bi-maximal mixings where [5]

$$\begin{aligned} U_{\rm MNS} &\simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \\ \Delta m^2_{\rm atm} &\simeq (1.6 \div 4) \times 10^{-3} {\rm eV}^2, \\ \Delta m^2_{\odot} &\simeq 10^{-2} \Delta m^2_{\rm atm}. \end{aligned} \tag{30}$$

The solar neutrino parameter $\Delta m^2_{\odot}$ is realized by the LMA–MSW scenario of neutrino oscillations.

We take

$$m_{\rm D}^{(1)} = U_{\rm MNS}\, {\rm diag}\,(0, \quad 10^{-11}, \quad 8 \times 10^{-10}\,). \tag{31}$$

Other parameters are the same as in Example II. This sort of additional contribution to the Dirac mass $m_{\rm D}$ does not affect the heavy neutrino sector. The result is

$$m_{\rm diag} \simeq {\rm diag}\,(0, 2 \times 10^{-11}, 6 \times 10^{-11}, 100, 151, 201) [{\rm GeV}], \tag{32}$$

$$\begin{aligned} U^{\rm Ll} &\simeq \begin{pmatrix} 0.577 - 0.01\,i & -0.706 - 0.006\,i & 0.003 + 0.405\,i \\ -0.577 + 0.01\,i & 0.001 & 0.007 + 0.814\,i \\ 0.577 + 0.01\,i & 0.706 + 0.018\,i & -0.01 + 0.409\,i \end{pmatrix}, \\ U^{\rm LH} &\simeq \begin{pmatrix} 0.03 & -0.048\,i & 0.036 \\ 0.03 & -0.048\,i & 0.036 \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{33}$$

The masses give appropriate $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\odot}$ (Eq. (30)). Large mixings in the $U^{\rm Ll}$ sector are obtained. Of course, this matrix is not unitary and differs from $U_{\rm MNS}$ in Eq. (30), the effect expected as heavy neutrino states affect the light sector.

Finally let us comment on the $\mu \to e\gamma$ decay. Analyzes of experimental data give [5]

$${\rm BR}(\mu \to e\gamma) = \frac{3\alpha}{8\pi} \left| \sum_i (U^{\rm LH})_{ei} (U^{\rm LH})^\dagger_{i\mu} \frac{M_i^2}{M_W^2} \right|^2 \leq 4.9 \times 10^{-11}. \tag{34}$$

In Eq. (34) contributions from the light neutrinos have been safely neglected [14]. Taking into account Eq. (33), ${\rm BR}(\mu \to e\gamma) \simeq 1.4 \times 10^{-12}$. It fits to the present limit.

3. TeV neutrino mass models without fine-tuning problems and symmetry arguments

Is it possible to avoid the problem of strong correlations among $m_{\rm D}$ and $M_{\rm R}$ elements (Eq. (23) and Eq. (25))? As discussed in [27], $m_{\rm D}$ and $M_{\rm R}$ originate from apparently disconnected mechanisms of gauge symmetry breaking of the SU(2)× U(1) gauge group and some larger unification group, respectively. Thus, it is hard to believe that these are arranged to fulfill Eq. (25) just to give TeV neutrinos with large LH mixings. In [27] a phenomenological model of the matrix M_ν with uncorrelated $m_{\rm D}$ and $M_{\rm R}$ matrix elements which realizes bi-maximal neutrino mixing has been constructed. The result discussed explicitly in [27] is the following

$$m_{\rm D} = m_3 \begin{pmatrix} x^2 y & 0 & 0 \\ 0 & x & x \\ 0 & -x^2 & 1 \end{pmatrix}, \tag{35}$$

$$M_{\rm R}^{-1}(x=0) = \frac{1}{M} \begin{pmatrix} 0 & 0 & \alpha \\ 0 & 1 & 0 \\ \alpha & 0 & 0 \end{pmatrix}, \tag{36}$$

where m_3 and M are of the order of the top quark and grand unification energy scale, respectively. $x = \mathcal{O}(m_c/m_t)$ is of the order of 10^{-2} (the ratio of the charm and the top quark mass), $y \simeq 10^{-1}$, $\alpha < 1$. The relation $x = 0$ in Eq. (36) stresses that $M_{\rm R}$ is independent of x being an element of $m_{\rm D}$. Null matrix elements in Eq. (35) and (36) are higher order powers in x and y and are neglected. This model has been originally used in the context of see-saw models. To accommodate it to TeV neutrinos, the M scale must be lowered to the TeV level. Then masses of light neutrinos of the order of $x^2 m_3^2/M$ appear (Eq. (31) in [27]) and, as in the previous section the fine-tuning problem shows up: the numerator must be tuned to fit light neutrino masses at the eV level. Moreover, using Eq. (5) we get

$$U^{\rm LH} \simeq \frac{m_3}{M} \begin{pmatrix} 0 & 0 & 0 \\ x\alpha & x & 0 \\ \alpha & -x^2 & 0 \end{pmatrix}. \tag{37}$$

We can see that these mixings are negligible. There is no contribution to the neutrino-less double beta decay and to the $\mu \to e\gamma$ decay from heavy neutrino mixing.

Let us finally comment on the possible symmetry of the full matrix M_ν. We note that the relations in Eq. (23) and (25) are not symmetries of M_ν but rather they are fine-tuning relations of elements of $m_{\rm D}$ and $M_{\rm R}$ what ensures that appropriate masses of light neutrinos can be obtained simultaneously with TeV neutrinos. Symmetries act directly on M_ν and not on objects which are functions of elements of M_ν. As discussed in the Introduction (and Example I), TeV neutrinos may lead to large LH mixings, but too large masses of light neutrinos would simultaneously arise. A source of the problem lies in a different scale of the elements of M_ν. Could symmetry of M_ν be able to reconcile the problem? Let us consider a toy model with only light (ν) and heavy (N) neutrinos. Let us assume that in the $(\nu, N)^{\mathrm{T}}$ basis the neutrino mass matrix is (elements a, b, c are real numbers)

$$M = \begin{pmatrix} a & b \\ b & c \end{pmatrix} . \tag{38}$$

The masses and a mixing angle are given by

$$m_{1,2} = \frac{1}{2}\left(a + c \mp \sqrt{(a-c)^2 + 4b^2}\right), \tag{39}$$

and

$$\sin 2\xi = \frac{2b}{\sqrt{(a-c)^2 + 4b^2}} . \tag{40}$$

If $c \gg b$, a then we get $|m_1| \simeq b^2/c$, $m_2 \simeq c \gg |m1|$ and $\xi \simeq b/c$. It is just a see-saw mechanism. If, however, $ac = b^2$ (due to symmetry!) then $m_1 \simeq 0$, $m_2 = a + c$ and $\sin 2\xi = 2\sqrt{ac}/(a+c)$. We can see that $\sin 2\xi \simeq 1$ if $a \simeq c$, which is, however, not a natural assumption. The problem does not vanish with larger dimension of M_ν. To summarize, a difficulty to build a symmetry of the M_ν matrix lies in the following: large LH mixings means that elements of the M_ν matrix are comparable. However, this is not true as long as the relation $m_{\mathrm{D}} \ll M_{\mathrm{R}}$ holds.

4. Conclusions

In summary, it has been shown that the present data from the light neutrino sector, especially their masses allow to construct the neutrino mass matrix M_ν with both TeV neutrinos and large LH mixings. However, the form of M_ν is constrained very much. In all three numerical examples of Section 2, LH mixings fulfill Eq. (5). The kind of relations (23) and (25) do not change this fact. There is no way around as long as $m_{\mathrm{D}} \ll M_{\mathrm{R}}$ which is, as discussed in Example I, a condition which must be fullfiled for our purposes.

The natural decomposition into $m_{\rm D}^{(0)}$ and $m_{\rm D}^{(1)}$ terms in Eq. (20) which has been used in this paper has been for the first time introduced in [11]. At this time an information on neutrino masses was completely different. Muon and tau neutrino masses at the level of keV and MeV, respectively has been allowed. Then some freedom of parameters in relations (23) and (25) was possible.

To avoid fine tuning problems we could look for a symmetry of the full matrix M_ν or build models with uncorrelated $m_{\rm D}$ and $M_{\rm R}$ matrices. However, as argued in Section 3, in the first case a kind of internal contradiction between a requirement of two different scales $m_{\rm D} \ll M_{\rm R}$ and large LH mixings arises. In the second case negligible LH mixings emerge.

The basic conclusions of the paper remain true regardless of the number of heavy neutrino states. The case when left-handed fields give a Majorana mass term $M_{\rm L}$ in Eq. (1) [9, 26] come to the same class of basic fine-tuning problems.

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